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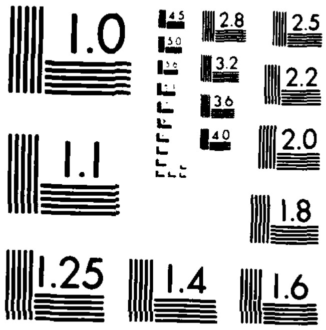

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STABLE MEASURES AND PROCESSES IN STATISTICAL PHYSICS

by

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and

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STABLE MEASURES AND PROCESSES IN STATISTICAL PHYSICS[1]

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Abstract. It is shown how α-stable distributions arise in statistical physics. A probabilistic proof of Khalfin's formula for decaying quantum systems is given. Also ergodic properties of symmetric α-stable flows in classical statistical mechanics are discussed.

1. Introduction.

Exactly sixty years ago Lévy (1924) has initiated the theory of stable distributions. His theory was completed by Gnedenko and Kolmogorov (1954), who mentioned "*it is probable that the scope of applied problems in which stable distributions play an essential role will become in due course rather wide*". The double anniversary motivated us to present some applications of stable distributions and processes to statistical physics. In passing, let us remark that in probability books only reference to Holtsmark (1915) work on the gravitational field of stars (3/2-stable distribution) is made. The only exception is a very recent book of Zolotarev (1983).

In recent years, inverse power long tails have become more evident in the analysis of physical phenomena and therefore stable distributions provide useful models. In the rest of this section we mention a number of works in this area. In section 2 we show how completely asymmetric α-stable distributions arise in quantum statistical physics and we sketch a probabilistic proof of Khalfin's formula for non-decay probability function. In section 3 we discuss how recent results of Cambanis, Hardin and Weron (1984) on ergodic properties of stationary symmetric α-stable processes can be used

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in classical statistical mechanics.

Let us recall that a probability distribution μ on $(-\infty,\infty)$ is α-stable if its characteristic function $\hat{\mu}(t) = \phi(t)$ is given by

$$\operatorname{Log} \phi(t) = \begin{cases} i\gamma t - (\sigma|t|^{\alpha}) \{1-i\beta\, \operatorname{sign}(t) \tan(\pi\alpha/2)\} & \text{if } \alpha \neq 1 \\ i\gamma t - \sigma|t| - i\beta(2/\pi)\sigma t \log|t| & \text{if } \alpha = 1, \end{cases} \tag{1}$$

where α,β,γ and σ are real constants with $\sigma \geq 0$, $0 < \alpha \leq 2$ and $|\beta| \leq 1$. See, Gnedenko and Kolmogorov (1954). Here α is the characteristic exponent, γ and σ determine location and scale. The coefficient β indicates whether the α-stable distribution is symmetric ($\beta=0$) or completely asymmetric ($|\beta| = 1$). Only in the case $0<\alpha<1$ the α-stable densities with $|\beta| = 1$ are one-sided i.e., their support is $[0,+\infty)$ for $\beta = 1$ and $(-\infty,0]$ for $\beta = -1$.

For a comprehensive survey of the recent works on α-stable processes and their relation via "correspondence principle" with α-stable measures on vector spaces cf. Weron (1984).

There are many physical phenomena which exhibit both space and time long tails and thus seem to violate the requirement of Gaussian distribution as a limit in the traditional central limit theorem. However, since these physical systems are stable in the sense a Gaussian is but without second moments, one suspects the use of stable distributions which have long tails to be relevant in the physics of these phenomena. A clear physical basis is required to justify the use of stable distribution in much the same way Khinchin (1949) gave a physical justification for the use of Gaussian distributions. Tunaley (1972) invoked physical arguments to suggest that if the frequence distributions in metallic films are stable then the observed noise characteristics in them may be understood. Based only on the experimental observations that near second order phase transitions where long tail spatial order develops, Jona-Lasinio (1975) considered stable distributions as a basic ingredient in understanding renormalization group notions in explaining such phenomena. Scher and Montroll (1975) connect intermittant currents in certain xerographic films to a stable distribition of waiting times for the jumping of charges out of a distribution of deep traps. This provided a basic theoretical model for dispersive transport in amorphous materials.

As examples of the exploration of the stable processes models in physical contexts, we may cite a few very interesting papers. Doob

(1942), West and Seshadri (1982) examined the response of a linear system driven by stable fluctuations. Mandelbrot and van Ness (1968) used Gaussian and stable fractional stochastic processes in several interesting situations. Montroll and West (1979), see also references there, Hughes, Shlesinger, and Montroll (1981), and Montroll and Shlesinger (1982) examined random walks with self-similar clusters leading to "Lévy flights" and "1/f noise". If the diffusion of defects in a medium containing many polar molecules is executed as a continuous-time random walk composed of an alternation of steps and passes and the pausing-time distribution function has a long tail, then Montroll and Bendler (1984) have obtained the Williams-Watts form of dielectric relaxation.

2. Decay theory of quantum systems.

The quantum description of decaying systems has been the subject of many investigations since the early days of quantum mechanics. For a review of different attempts to solve this interesting problem of quantum physics, see, Fonda et al, (1978). Let us recall only that the discovery of natural radioactivity Becquerel (1896) and the identification of two new radioactive elements, i.e., polonium and radium by Mme Sklodowska-Curie (1898) marks the beginning of the studies of decay processes. The classical theory of the decay is based on the assumption that radioactive nuclei have a certain probability of undergoing decay and that this probability does not depend on the past history of the individual decaying nuclei. From which one gets the exponential decay law

$$N(t) = N(0) \exp(-t/\tau), \tag{2}$$

where $N(t)$ is the number of radioactive nuclei which are present at time t and τ is the lifetime of a radioactive nucleus.

In quantum description of the decay process, one determines the probability $P(t)$ of finding, for a measurement at time t, the quantum system in the same physical situation, i.e., in the same state ψ, in which it was at the initial time $t=0$. The mathematical quantity which is relevant for this problem is then

$$P(t) = |A(t)|^2, \qquad t \geq 0 \tag{3}$$

where

$$A(t) = (\psi, \exp(-Dt/\hbar)\psi), \tag{4}$$

D being the development operator governing the dynamical evolution of the quantum system under investigation, and $h = 2\pi\hbar$ is Planck's constant. Originally, Krylov-Fock (1947) used in formula (4) Hamiltonian operator L, but their model is not now accepted, see Fonda et al. (1978), since it is known that a decaying system cannot be described by the unitary evolution $U_t = \exp(-iLt)$.

When an ensemble of identical quantum systems is considered, the number N(t) of systems which are found in the original state at time t is given by

$$N(t) = N(0)P(t), \tag{5}$$

Equation (5) is then the quantum analogue of the classical equation (2). Several authors have studied in various contexts the behaviour of P(t).

We shall derive the nonexponential form of P(t) for many-body systems from a completely asymmetric α-stable energy distribution of the decaying system. Let us mention that nonexponential decay law obtained first asymptotically for large times by Khalfin (1957), still attracts interest, see for example Bunimovich-Sinai (1981), Hack (1982), Lee (1983), and Hart-Girardeau (1983).

THEOREM 1.

The non-decay probability function for many-body weakly interacting quantum systems has the form

$$P(t) = \exp(-ct^{\alpha}), \qquad c > 0,\ 0 < \alpha < 1. \tag{6}$$

Proof. In the quantum statistical mechanics the time evolution of a physical system in equilibrium is given by a dynamical group $U_t = \exp(-iLt)$, which is uniquely determined by its generator defined by the Hamiltonian L of the system. In order to handle a decaying system this time evolution has to be generalized, since the decaying systems are not relevant to the discussion of equilibrium.

The time evolution of decaying system is described by dynamical semigroup cf. Davis (1976), Fonda et al. (1978), and Blum (1981). For this consider a continuous one-parameter semigroup T_t of contractions on the Hilbert space L(H) of all Hilbert-Schmidt operators on the Hilbert space H associated with a quantum mechanical system.

By Nagy-Foias (1960) theorem this semigroup uniquely splits into the orthogonal sum of a unitary semigroup and of a completely non-unitary (c.n.u.) semigroup

$$T_t = T_t^u \oplus T_t^{cnu}.$$

Hille-Yosida theorem gives a form of infinitesimal generators

$$T_t^u = \exp\{(-it\tilde{L})/\hbar\}$$

and

$$T_t^{cnu} = \exp\{(-t\tilde{D} - it\tilde{L}_1)/\hbar\},$$

where $\tilde{L}$, $\tilde{L}_1$, $\tilde{D}$ are self-adjoint operators on L(H) and $\tilde{D}$ has positive spectrum.

If $\rho(t)$ is a density operator of the system i.e., self-adjoint, positive with finite trace (see Blum (1981)), then $\rho(t) = T_t\rho(o)$ and

$$\rho(t) = (T_t^u \oplus T_t^{cnu})\ \rho(t) = \rho^u(t) \oplus \rho^{cnu}(t).$$

Moreover,

$$i\hbar \frac{d}{dt}\rho^u(t) = \tilde{L}\ \rho^u(t) = [L, \rho^u(t)] \tag{7}$$

and

$$i\hbar \frac{d}{dt}\rho^{cnu}(t) = (\tilde{L}_1 - i\tilde{D})\ \rho^{cnu}(t) = [L_1, \rho^{cnu}(t)] - [D, \rho^{cnu}(t)]_+ \tag{8}$$

where L, L_1, D are self-adjoint operator on H, L, L_1 are Hamiltonians and D a new development operator with positive spectrum. Formula (7) is a classical von Neumann equation and (8) its analogue for c.n.u. part. [,] denotes here comutator and $[\ ,\ ]_+$ anticomutator, for more details cf. Weron, Rajagopal, and Weron (1984).

In particular, assuming for simplicity that $\tilde{L}_1 = 0$, we have

$$\rho^{cnu}(t) = e^{-t\tilde{D}/\hbar}\rho^{cnu}(0) = e^{-tD/\hbar}\rho^{cnu}(0)e^{-tD/\hbar}, \tag{9}$$

which shows that D governs the dynamical evolution of the decaying quantum system.

Introducing probability density $p(\varepsilon)$ of the state ψ associated with the continuous spectrum of the development operator D one can write (see (3) and (4))

$$A(t) = (\psi, \exp(-Dt/\hbar)\psi) = \int_0^\infty \exp(-\varepsilon t/\hbar)\ (\psi, E(d\varepsilon)\psi) =$$

$$= \int_0^\infty \exp(-\varepsilon t/\hbar)\ p(\varepsilon)d\varepsilon, \tag{10}$$

where $E(\cdot)$ is the spectral measure of the development operator D. Thus A(t) is the Laplace transform of the probability density $p(\varepsilon)$ of the decaying state ψ.

Observe that there is an arbitrariness in the specification of ψ and $p(\varepsilon)$. In general one considers ψ to represent a decaying state for a many-body system, and therefore the number of components in the system should not influence the decay. In other words the same decaying law should be obtained for one portion or several portions of the system. Consequently, in a weakly interacting quantum system microscopic energies can be considered as independent identically distributed energy random variables. The macroscopic energy distribution $p(\varepsilon)d\varepsilon$ associated with the decaying system is identified to be the limit distribution of normalized sums of the microscopic energy random variables. By the limit theorem, Gnedenko and Kolmogorov (1954), it is well known that the limit $p(\varepsilon)d\varepsilon$ has α-stable distribution $0 < \alpha \leq 2$. Since $p(\varepsilon)$ is associated from the above construction with the development operator D, it has to have positive support. This holds only when $p(\varepsilon)d\varepsilon$ has a completely asymmetric ($\beta = 1$, $0 < \alpha < 1$) stable distribution. Thus by (10) it is enough to evaluate its Laplace transform. In the case at hand, formula (1) can be rewritten, if we put $\gamma = 0$, in the following form

$$\text{Log } \phi(t) = -\sigma_1 t^{\alpha} (\cos(\pi\alpha/2) - i \sin(\pi\alpha/2)) = -\sigma_1(-it)^{\alpha},$$

where $\sigma_1 = \sigma/\cos(\pi\alpha/2)$ and $t \geq 0$.

Consequently, the Fourier transform of $p(\varepsilon)d\varepsilon$ has the form $\exp(-\sigma_1(t/i)^{\alpha})$. By the well known relation between Fourier and Laplace transforms, $F(f(x);t) = L(f(x);-it)$, when f(x) has positive support, see Gradshteyn and Ryzhik (1980) p. 1153. Hence we get that the Laplace transform of $p(\varepsilon)d\varepsilon$

$$L(p(\varepsilon);t) = \exp(-\sigma_1 t^{\alpha}) \tag{11}$$

Finally, by (3), (4), (10) and (11)

$$P(t) = [\exp(-\sigma_1 t^{\alpha})]^2 = \exp(-2\sigma_1 t^{\alpha}),$$

which gives formula (6) with $c = 2\sigma_1$ and $0 < \alpha < 1$. □

3. Ergodic properties of stable dynamical systems.

According to the theory of ensembles, cf. Arnold and Avez (1968), an isolated system is in equilibrium when it is represented by a "microcanonical ensemble" i.e., when all points on the surface of given energy have the same probability. This means that the energy must be the only invariant. But for many physical systems energy is far from being the only invariant and consequently a system is wandering on a very small fraction of the constant energy.

Boltzmann introduced a new (ergodic) type of dynamics system for which the energy is the only invariant. In its modern form Boltzmann's Ergodic Hypothesis: "*the point of phase space representing the state of Hamiltonian systems wanders everywhere on its hypersurface of constant energy*" is replaced by the notion of metric transitivity. It says that every subset of a hypersurface of constant energy that is carried into itself by the time development of the system is either of measure zero or is the complement of a subset of measure zero.

The measure referred to is given by the so-called micro-canonical ensemble

$$\mu(S) = \int_S \delta(E - H(q,p))\,dq\,dp,$$

where S is a subset of the hypersurface of *given energy* E, $q = \{q_1,\dots,q_m\}$ and $p = \{p_1,\dots,p_m\}$ are the canonical variables and H is the Hamiltonian of the system. The time evolution of the system is given by a *flow* i.e., a *measure preserving family of mappings* $T^t\{q,p\} = \{q(t),\ p(t)\}$ such that

$$T^o = I,\ T^nT^k = T^{n+k},\ \mu(T^nS) = \mu(S),\quad n,k \in \mathbb{Z}.$$

Any flow induces a one parameter group of transformations of functions f's defined on the hypersurface Ω_E of given energy E:

$$X_n \equiv (U^n f)\ (q,p) \equiv f(T^n\{q,p\}),\quad n \in \mathbb{Z}.$$

Now a flow is *metrically transitive* if the only functions satisfying $U^n f = f$ for all n, except possibly on a set of μ-measure zero, are constant almost everywhere.

Observe that $\Omega_E \subset \mathbb{R}^{2m}$ with the micro-canonical measure μ plays a role of probability space adequate to our problem. Thus all probabilistic characteristics of the flow can be expressed in terms of the measure μ. See Cornfeld-Fomin-Sinai (1982).

When the functions f's are chosen in the Hilbert space $L^2(\Omega_E,\mu)$, for example when μ is Gaussian, then $\{U^n,\ n \in \mathbb{Z}\}$ turns out to be a unitary one-parameter group $U^nU^k = U^{n+k}$, $U^o = I$ and

$(U^n)^* = U^{-n}$. Von Neumann's ergodic theorem says that

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} f(T^n x)$$

exists in $L^2(\Omega_E, \mu)$ and equals

$$\int_{\Omega_E} f(x)\, d\mu\,(x),$$

i.e., time average equals phase average.

When the functions f's are chosen in the Banach space $L^p(\Omega_E, \mu)$, for example when μ is a symmetric α-stable measure and $1 \leq p < \alpha < 2$, then $\{U^n,\ n \in \mathbb{Z}\}$ becomes a group of isometries on $L^p(\Omega_E, \mu)$ and similar ergodic behaviour follows from Bellow's (1964) ergodic theorem.

Professional statistical mechanicians are not much impressed. They ask: *How does one verify that a concretely given dynamical system is metrically transitive?* The answer is that it isn't so easy to do. However, it is well known that there are many ergodic ($\equiv$ metrically transitive) flows and also many which are not ergodic, cf. Cornfeld, Fomin and Sinai (1982), where Gaussian case is studied in detail. Since α-stable distributions form a universal class of limit distributions, the systematic use of limit theorems for rigorous proofs in statistical mechanics originated by Khinchin (1949), motivated us to study symmetric α-stable flows.

In order to define them consider the space S of all real sequences $\{x(n),\ n \in \mathbb{Z}\}$ with the minimal σ-algebra A containing all the finite-dimensional cylinders. The probability measure m on A is said to be symmetric α-stable if the joint distribution of any vector

$$X = (x(n_1),\ x(n_2), \ldots, x(n_r))$$

is an r-dimensional symmetric α-stable distribution i.e., if its characteristic function has a form

$$\exp\left(-\int_{S_r} |\langle t,x\rangle|^{\alpha} d\Gamma_X(dx)\right),$$

where $t = (t_1, \ldots, t_r)$, $x = (x_1, \ldots, x_r) \in \mathbb{R}^r$, $\langle t,x\rangle = t_1x_1 + \ldots + t_rx_r$, and Γ_X is a symmetric finite measure on the unit sphere S_r of $\mathbb{R}^r$ - called the spectral measure of the vector X. Denote by T^n the shift transformation in the space S, i.e., $T^n x(k) = x(n+k)$. If the measure m is invariant w.r.t. T^n then the group $\{T^n, n \in \mathbb{Z}\}$ of shifts on the space S is said to be symmetric α-stable flow.

As it follows from Hardin (1982) any such flow (or equivalently its induced group U^n of transformations) can be represented in law by

$$X_n = \int_M (U^n f)(x) Z(dx), \tag{12}$$

where (M,Σ,ν) is a measure space, $f \in L^\alpha(M,\Sigma,\nu) \equiv L^\alpha(\nu)$ is a fixed function, $\{U^n, n \in \mathbb{Z}\}$ is a group of isometries on $L^\alpha(\nu)$ is the canonical independly scattered symmetric α-stable measure on (M,Σ,ν) i.e., for all disjoint sets $M_1,\ldots,M_n \in \Sigma$ of finite ν-measure the random variables $Z(M_1),\ldots,Z(M_n)$ are independent with

$$\mathbb{E} \exp(itZ(M_k)) = \exp(-|t|^\alpha \nu(M_k)).$$

Observe that a mean zero Gaussian ($\alpha = 2$) flow on (Ω_E,μ) can be trivially represented in form (12). To see this, let Z be the canonical independently scattered Gaussian measure on (Ω_E,μ). Then by checking characteristic functions we see that the Gaussian flow $U^n f$ has the same distribution as

$$\int_{\Omega_E} U^n f(x) Z(dx)$$

The following general answer for the above discussed question can be immediately obtained from the recent result of Cambanis, Hardin and Weron (1984) on stationary stable processes.

THEOREM 2.

A symmetric α-stable flow, $0 < \alpha \leq 2$ with the spectral representation (12) is metrically transitive iff for each $h \in \overline{sp}\{U^n f, n \in \mathbb{Z}\}_{L^\alpha(\nu)}$ *the following two conditions hold*

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} ||U^n h - h||_\alpha^\alpha = 2||h||_\alpha^\alpha . \tag{13}$$

and

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} ||U^n h - h||_\alpha^{2\alpha} = 4||h||_\alpha^{2\alpha}. \tag{14}$$

It turns out that these conditions provide a useful criterion for metric transitivity. For example moving average α-stable flows are metrically transitive and α-sub-Gaussian flows are never metrically transive. For the proof of Theorem 2 and more results we refer to Cambanis, Hardin, and Weron (1984). Here we will discuss only one example.

EXAMPLE

A real symmetric α-stable flow is called *harmonizable* if its induced group of transformations $X_n = U^n f$ has the following representation:

$$X_n = \mathrm{Re} \int_0^{2\Pi} e^{in\lambda} dW(\lambda), \quad n \in \mathbb{Z}, \tag{15}$$

where W is a rotationally invariant, i.e., the distribution of $\{e^{iv} W(\Delta),\ \Delta \in B[0,2\pi)\}$ does not depend on v, independently scattered complex symmetric α-stable measure on $([0,2\pi), B, \nu)$ and ν is finite, see Cambanis (1983). Of course, any real Gaussian flow has a harmonic representation (15), where $W(\cdot)$ is an independently scattered complex Gaussian measure such that $\mathbb{E} \exp\{i \,\mathrm{Re} \int u dW\} = \exp(-||u||^2)$, $u \in L^2(\nu)$. However, α-stable flows with $0 < \alpha < 2$ do not have in general harmonic representation cf. Cambanis and Soltani (1984).

It is well known, Maruyama (1949), Grenander (1950), and Fomin (1950), that a Gaussian flow is metrically transitive iff the spectral measure F $(F(\Delta) = \mathbb{E}|W(\Delta)|^2)$ has no atoms. For $\alpha<2$ it is enough to check conditions (13) and (14) in Th. 2. Note that the left hand side of the formula (13) takes the form

$$\frac{1}{N} \sum_{n=1}^{N} ||(e^{in\lambda}-1)h(\lambda)||_\alpha^\alpha = \frac{1}{N} \sum_{n=1}^{N} \int_0^{2\pi} |e^{in\lambda}-1|^\alpha \ |h\ (\lambda)|^\alpha \nu(d\lambda) =$$

$$= \frac{2^\alpha}{N} \int_0^{2\pi} \sum_{n=1}^{N} |\sin \frac{n\lambda}{2}|^\alpha \ |h(\lambda)|^\alpha \nu(d\lambda)$$

$$\underset{N\to\infty}{\to} 2^\alpha/\pi \int_0^\pi |\sin x|^\alpha dx \int_{(0,2\pi)} |h(\lambda)|^\alpha \nu(d\lambda) \equiv C_\alpha ||h||_\alpha^\alpha$$

Observe that when $\alpha = 2$, $C_2 = 2$ and thus (13) holds provided $\nu\{0\}=0$. However, when $\alpha < 2$ then $C_\alpha < 2$ and (13) is not satisfied. Consequently by Th. 2 symmetric α-stable harmonizable flow is *never metrically transitive* for $\alpha < 2$. This fact has been established also by LePage (1980) by a different method. □

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